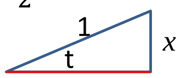
Integraalrekening (voor gevorderden)

Uitkomsten:

1. $\int \frac{dx}{\sqrt{16-x^2}} = \arcsin\left(\frac{x}{4}\right) + c$
2. $\int \frac{3dx}{25+x^2} = \frac{3}{5}\arctan\left(\frac{x}{5}\right) + c$
3. $\int \frac{8dx}{x\sqrt{4x^2-1}} = 8\,\mathrm{arcsec}(2x) + c$
4. $\int \frac{x}{\sqrt{1-x^4}}dx = \frac{1}{2}\arcsin(x^2) + c$
5. $\int \frac{x}{x^4+36}dx = \frac{1}{12}\arctan\left(\frac{1}{6}x^2\right) + c$
6. $\int \frac{x^3}{x^2+1}dx = \int\left(x - \frac{x}{x^2+1}\right)dx = \frac{1}{2}x^2 - \frac{1}{2}\ln(x^2+a) + c$
7. $\int \frac{e^{3x}}{9+e^{6x}}dx = \frac{1}{9}\arctan\left(\frac{1}{3}e^{3x}\right) + c$
8. $\int \frac{x-3}{x^2+1}dx = \frac{1}{2}\ln(x^2+1) - 3\arctan(x) + c$
9. $\int \frac{dx}{x^2-4x+7} = \int \frac{dx}{(x-2)^2+3} = \frac{1}{3}\sqrt{3}\arctan\left(\frac{x-2}{\sqrt{3}}\right) + c$
10. $\int \frac{xdx}{\sqrt{9+8x^2-x^4}} = \int \frac{xdx}{25-(x^2-4)^2} = \frac{1}{2}\arcsin\left(\frac{x^2-4}{5}\right) + c$
11. $\int \sqrt{1-x^2}\,dx = \int cos^2(t)dt = \int(\frac{1}{2} + \frac{1}{2}\cos(2t)\,dt =$

    $=\frac{1}{2}arcsin(x) + \frac{1}{2}x\sqrt{1-x^2} + c$

12. $\int \sqrt{4-x^2}\,dx = 2\arcsin\left(\frac{1}{2}x\right) + \frac{1}{4}x\sqrt{1-\frac{1}{4}x^2} + c$
13. $\int \sqrt{1-4x^2}\,dx = \frac{1}{4}\arcsin(2x) + \frac{1}{2}x\sqrt{1-4x^2+c}$
14. $\int \frac{dx}{x^2\sqrt{x^2+4}} = \frac{-\sqrt{x^2+4}}{4x} + c$
15. $\int \sqrt{x^2-1}\,dx = \frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2}ln\left|x+\sqrt{x^2-1}\right| + c$

    Deze opgave is moeilijk!

subst. van $x = \sec(u)$ geeft: (onder de wortel: $\frac{sin^2(u)}{cos^2(u)}$) en met

$x = \frac{1}{\cos(u)} = (\cos(u))^{-1}$ wordt: $dx = \frac{\sin(u)}{cos^2(u)}$ en de splitsing van

$sin^2(u) = 1 - cos^2(u)$ geeft dan $\int \frac{du}{cos^3(u)} - \int \frac{du}{\cos(u)}$ . . . .

Kijk uit voor schrijffouten . . .een mintekentje ergens vergeten?...

en kijk nog eens terug naar de theorie!

16. $\int \frac{dx}{x^2\sqrt{x^2-9}} = \frac{\sqrt{x^2-9}}{9x} + c$

17. $\int \frac{dx}{x^2\sqrt{25-x^2}} = \frac{-1\sqrt{25-x^2}}{25x} + c$

18. $\int \frac{x}{\sqrt{x^2+4}}\ dx = \sqrt{x^2+4} + c$ "lekker" makkelijk!

19. $\int \frac{x^3}{\sqrt{x^2+9}}\ dx = \frac{1}{3}(x^2-18)\sqrt{x^2+9} + c$

20. $\int x\sqrt{1-x^4}\ dx = \frac{1}{4}\arcsin(x^2) + \frac{1}{4}x^2\sqrt{1-x^4} + c$

21. $\int \frac{dx}{\sqrt{x^2+16}} =$ ...$=\ln\left(x+\sqrt{x^2+16}\right) + \boldsymbol{c}$ ... in die $\boldsymbol{c}$ zit $\ln(4)$ "verstopt"

22. $\int \sqrt{5+4x-x^2}\ dx = \int \sqrt{9-(x-2)^2}\ dx =$

$\frac{9}{2}\arcsin\left(\frac{1}{3}(x-2)\right) + \frac{1}{2}(x-2)\sqrt{5+4x-x^2} + c$

23. $\int \frac{dx}{\sqrt{9x^2+6x-8}} = \frac{1}{3}\,ln\left|3x+1+\sqrt{9x^2+6x-8}\right| + \boldsymbol{c}$

en hier zit in de constante iets verstopt: "de 3 van binnen de ln"

24. $\int \sqrt{1+x^2}\ dx = \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}ln\left|x+\sqrt{1+x^2}\right| + c$

Deze opgave sluit aan/af op de theorie: heb je dit goed en onder de knie?
Nu met integratiegrenzen erbij:

25. $\int_0^{\frac{1}{2}} \sqrt{1-2x}\ dx = \frac{1}{3}$

26. $\int_1^e \frac{x+1}{x}\ dx = e$

27. $\int_0^{\frac{1}{2}} \frac{-1}{\sqrt{1-4x^2}}\ dx = -\frac{1}{4}\pi$

28. $\int_0^3 \frac{x^2-3x}{x+1}\ dx = 4\ln(4) - 7\frac{1}{2}$ . . . uitdelen!

29. $\int_1^2 2^{-x-1}\ dx = \ldots = \frac{1}{8\ln(2)}$

30. $\int_0^1 x\sqrt{4-x^2}\ dx = \ldots = \frac{8}{3} - \sqrt{3}$

31. $\int_0^{\pi} \frac{3\sin(x)}{2-\cos(x)}\ dx = 3\ln(3)$

32. $\int_{\ln(2)}^{\ln(1+e)} \frac{e^{2x}}{2e^x-2} = \left[\frac{1}{2}e^x - \frac{1}{2}ln|2e^x-2|\right]_{\ln(2)}^{\ln(1+e)} = \ldots = \frac{1}{2}e - 1$

33. $\int_0^5 \frac{x}{x^2+10}\ dx = \ldots = \frac{1}{2}\ln(7\frac{1}{2})$

34. $\int_0^{\frac{1}{2}\pi} \frac{\sin(x)}{(1+\cos(x))^2}\ dx = \left[\frac{1}{1+\cos(x)}\right]_0^{\frac{1}{2}\pi} = 1 - \frac{1}{2} = \frac{1}{2}$

35. $\int_2^3 \frac{dx}{16-8x+x^2} = \frac{1}{2}$

36. $\int_0^{\frac{1}{4}\pi} \frac{1}{2}\tan(x)\,dx = \frac{1}{4}\ln(2)$

37. $\int_0^1 \frac{4e^x}{1+e^{2x}}\,dx = 4\arctan\left(\frac{e}{1+e^2}\right) - 4\arctan(\frac{1}{2})$

38. $\int_1^e \frac{2+2\ln(x)}{x}\,dx = 3$

39. $\int_3^5 \frac{|x-4|}{\sqrt{x}}\,dx = \int_3^4 \frac{4-x}{\sqrt{x}}\,dx + \int_4^5 \frac{x-4}{\sqrt{x}}\,dx = \cdots = 21\frac{1}{3} + 10\sqrt{3} - 4\frac{2}{3}\sqrt{5}$

40. $\int_0^1 x^2 e^{(-x^3)}\,dx = \frac{1}{3} - \frac{1}{3e}$

41. $\int_2^3 \frac{x^2}{1-x}\,dx = -3\frac{1}{2} - \ln(2)$

42. $\int_1^2 \frac{e^x}{1-e^x}\,dx = \ldots = -\ln(e+1)$

43. $\int_{-3}^{-1} \frac{dx}{x^2+4x+5} = -\frac{1}{2}\pi$

44. $\int_0^{\frac{1}{4}\pi} \frac{tan^5(x)}{cos^2(x)}\,dx = \left[\frac{1}{6}tan^6(x)\right]_0^{\frac{1}{4}\pi} = \frac{1}{6}$

45. $\int_0^1 \frac{2x}{(x+1)^2}\,dx = 2\ln(2) - 1$

Tot slot de “oneigenlijke integralen”:

46. $\int_0^\infty x e^{-\frac{1}{2}x^2}\,dx = \lim_{t\to\infty}\left[-e^{-\frac{1}{2}x^2}\right]_0^t = 1$

47. $\int_0^1 \frac{2x}{\sqrt{1-x^4}}\,dx = \lim_{t\uparrow 1}[\arcsin(x^2]_0^t = \frac{1}{2}\pi$

48. $\int_{-\infty}^0 \frac{dx}{1+e^{-x}} = \cdots = \ln(2)$

49. $\int_0^\infty \frac{dx}{2e^x} = \cdots = \frac{1}{2}$

50. $\int_0^\infty \frac{5}{1+x^2}\,dx = 2\frac{1}{2}\pi$

51. $\int_1^\infty \frac{5}{3x\sqrt{x}}\,dx = \ldots 3\frac{1}{3}$

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